STRAIN LOCALIZATION AT FINITE STRAINS: MATHEMATICAL MODELING AND NUMERICAL IMPLEMENTATION

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Materials such as metals, soils and rocks exhibit regions of highly localized strains – commonly referred to as shear bands – when deformed into their plastic regime. Although shear banding is one of many possible deformation modes, it is usually a precursor to more or less catastrophic failures.

The physical phenomena responsible for localization can vary widely and are sometime difficult to isolate. Lack of homogeneity, strain rates and other causes can trigger localized deformations.

Strain localization has been observed in geomaterials in a variety of situations. For instance, triaxial tests in the laboratory and excavation sites in the field have provided observations of localized deformations. Gaining a better understanding of the mechanics and physics of shear banding is important for design, exploration, and exploitation purposes.

Mathematical and numerical modeling is an invaluable complement to experimental investigations in the solution to most engineering problems. The body of literature on the modeling of localization in geomaterials has been growing rapidly, especially during the last two decades, with most of the emphasis on detecting the onset of instability under quasi-static and small strains conditions, and predicting the orientation of the shear band. We propose to investigate the effects of geometric nonlinearities and the post-localization behavior from a mathematical point of view. The results will then be implemented with appropriately constructed finite elements (capable of efficiently capturing discontinuities) in a numerical simulation of laboratory experiments and field observations.

With a rate independent constitutive model describing the behavior of the material, the analysis seeks a point in the loading program where the equilibrium equations become ill-posed. This bifurcation point is generally signaled by the singularity of the so-called acoustic tensor at points located within a certain band oriented in some optimum direction. Some investigators have reported that considering finite strains (instead of the more common small strain theory) can drastically affect when and how localization occurs; thus the importance of incorporating geometric nonlinearities when justified. Our investigation will help improve the understanding of these large deformation effects.

Since the mathematical problem becomes ill posed at the onset of localization, some modifications have to be made in order to pursue the loading program further. Regularization techniques using viscoplastic, gradient-dependant or non local models have been used to circumvent this problem in the contest of weak discontinuities where the shear band has a finite width. For strong discontinuities (shear bands of zero thickness), it has been shown that the hardening modulus must bifurcate (become singular at some points) for localization to occur, and theories pertaining to post-localization are not well-developed. This is another area where we intend to make a contribution by conducting comprehensive studies of possible post-localization scenarios and comparing our results with experimental data.

The findings of these investigations could lead to improvement of the current techniques used to analyze the stability of retaining walls, deformations around excavation zones, the process of rupture (faulting) in the earth's crust, etc. In addition these tools would be easily adapted to other types of materials (metals, polymers, etc.), where shear banding is known to occur.