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The John A. Blume Earthquake Engineering Center

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The Blume Center Welcomes the New Year and a new Associate Director!

Happy New Year to all of you. The Blume Center and Stanford University survived the Y2K bug and we are rushing toward midterms for the Winter Quarter.

The Blume Center is pleased to announce that as of September 1999, Associate Professor **Gregory G. Deierlein** has been named Associate Director of the Blume Center. We welcome Prof. Deierlein and look forward to working with him to acheive the goals of the Blume Center.

921 Chi-chi Earthquake Pictures Now on Web

On September 21, 1999 a strong earthquake with local magnitude of 7.3 and moment magnitude of 7.7 shook the island of Taiwan. The ground shaking was triggered by the rupture of Chelungpu Fault over a length of about 80 km at a shallow depth of approximately 1 km in a small town named Chi-chi in the county of Nantou.

The "921 Chi-chi Earthquake," as it is now called, claimed over 2,300 lives, mostly in Nantou and Taichung counties, and caused over 8,700 injuries; economic losses from thousands of buildings and infrastructures destroyed by the earthquake are estimated to reach over US$10 billion.

About two weeks after the earthquake, Professors Ronaldo Borja and Laura Lowes, and Ph.D. candidate Chao-Hua (Eric) Lin, went to Taiwan to see the damaged sites in person. They brought back with them some of the most vivid images captured on camera of the devastation created by the earthquake, which are now on display at *http://blume.stanford.edu/*.

Funding for their travel was provided by The John A. Blume Earthquake Engineering Center and PEER (Pacific Earthquake Engineering Research Center).

BLUME CENTER NEWS

Wei-Ming Chi and **Sameh Mehanny** both filed their dissertations and received their Ph.D. degrees in January, 2000: **Chi** is currently a research engineer with General Electric and **Mehanny** is working as an engineer with the San Francisco office of Simpson, Gumpertz & Hegar, Inc.

Professor Gregory Deierlein was elected to "Fellow" in ASCE in recognition of professional acheivements and contributions as a structural engineeer and educator.

On October 2, Stanford hosted the **PEER Earthquake Engineering Scholar's Course** (Seismology) at The John A. Blume Earthquake Engineering Center. The Earthquake Engineering Scholar's Course (EESC) is a multi-campus program that provides instruction to undergraduate students during four weekend retreats at PEER core university campuses during the school year. EESC is intended for engineering seniors who have demonstrated a sincere interest in earthquake engineering or an earthquake-related field and have achieved a high level of scholarship.

In October, **Professor Anne Kiremidjian** gave a presentation at the CUREe/Kajima Phase III meeting in Tokyo, Japan on "Assembly Based Vulnerability for Buildings". She also presented a paper on "Vulnerability and Damageability of Building in Earthquakes" at the Workshop on "The Uncertainty of Damageability" organized by the Risk Prediction Initiative, Bermuda, in November.

Professor Ronaldo Borja participated in the International Workshop on Localization and Bifurcation in Geomechanics held in Perth, Australia from November 29-December 2, 1999. He gave a talk on modeling of strain localization in rocks based on strong discontinuity approach.

NEW PROJECT: "Seismic Performance of Gypsum Drywalls: Analytical Component"; Part of the CUREe-Caltech Woodframe Project supported by FEMA to assess and improve the seismic performance of woodframe buildings. Princial Investigator: **Gregory G. Deierlein**; Student Investigator: **Amit Kanvinde.**

RESEARCH SPOTLIGHT

EXTRACTION OF RITZ VECTORS USING A COMPLETE FLEXIBILITY MATRIX

By Hoon Sohn and Kincho H. Law

INTRODUCTION

Load-dependent Ritz (or Lanczos) vectors have gained much interest in structural dynamics. For the reduction of linear dynamic systems, Ritz vectors are shown to be more effective to approximate the response quantities of interest using a smaller number of independent basis vectors than modal vectors. Ritz vectors are also useful for finding partial extremal solutions of large eigenvalue problems and to reanalyze structural system with a few localized modifications. In structural monitoring and damage diagnosis, Ritz vectors have been shown to offer superior performance in identifying damage [4]. However, in experimental vibration tests, typically only modal parameters are extracted. It is only recently that Cao and Zimmerman has proposed a state-space-formulation to extract Ritz vectors from measured vibration tests [1].

In this study, a new extraction procedure for Ritz vectors is proposed. The procedure is based on a complete flexibility matrix, which is obtained from the dynamic test data. The proposed method enables to generate Ritz vectors from any assumedload patterns, and the contribution of unmeasured modes to the complete flexibility is explicitly considered in the extraction procedure.

EXPERIMENTAL EXTRACTION OF RITZ VECTORS

The extraction of Ritz vectors starts with the assumption that the dynamic loading $\mathbf{F}(t)$ can be separated into a spatial load vector $\mathbf{f}$ and a time function u(t):

$$\mathbf{F}(t) = \mathbf{f}\, u(t)$$

If the modal vectors are mass-normalized such that $V^T KV = O$ and $V^T MV = I$, the flexibility matrix can be expressed in terms of modal parameters:

$$\mathbf{G} = \mathbf{K}^{-1} = \mathbf{V}\mathbf{O}^{-1}\mathbf{V}^T$$

where Ω is the diagonal eigenvalue matrix and V is the corresponding eigenvector (modal vector) matrix.

In most experimental modal analyses, only a few lower modal frequencies and modal vectors are identified. For this case, the flexibility matrix is divided into the measured *modal flexibility*, which is formed from the estimated frequencies and modal vectors, and *residual flexibility* formed from the residual modes [2]:

$$\mathbf{G} = \mathbf{G}_m + \mathbf{G}_r = \mathbf{V}_m \mathrm{O}_m^{-1} \mathbf{V}_m^T + \mathbf{V}_r \mathrm{O}_r^{-1} \mathbf{V}$$

where the subscripts m and r denote the measured and residual quantities, respectively. Here, the modal flexibility is easily constructed using the measured natural frequencies and modal vectors. The residual flexibility is the contribution of the unmeasured dynamic modes to the full flexibility matrix. In general, the contribution of lower modes, which are normally estimated in experimental modal analyses, are more significant than those of higher modes because the contribution of each mode is inversely proportional to the magnitude of the corresponding natural frequencies. However, as shown in the example later, the contribution of the residual flexibility cannot be neglected in the extraction procedure of Ritz vectors.

The residual flexibility can be estimated using a residual function, which is computed by subtracting the reconstructed response of the identified modes from the measured frequency response functions (FRFs) :

$$\begin{aligned}\mathbf{R}(?) &= \mathbf{H}(?) + ?^2 \mathbf{V}_m \left(\mathbf{O}_m - ?^2 \mathbf{I}\right)^{-1} \mathbf{V}_m^T \\ &= -\mathbf{G}_r\, ?^2\end{aligned}$$

Curve-fitting over a set of frequency samples yields an estimate of the residual flexibility:

$$\mathbf{G}_r = -\sum_{w \in w_{set}} \frac{\mathbf{R}(w)}{w^2}$$

In this study, $\mathrm{H}(w)$ and $\mathrm{R}(w)$ are assumed to be squared matrices. That is, individual modal testing is conducted by applying an impulse excitation force to a single degree of freedom (DOF) and the excitation is repeated for all measured DOFs. Reference [2] addresses the extraction of a complete flexibility when the number of excitation points is less than that of response points. Although it is not shown here, the formulation of the residual flexibility, can be easily modified to include damping.

From the measured flexibility matrix **G** (=+) and the analytical mass matrix **M,** the first Ritz vector can be computed as:

$$\bar{\mathbf{r}}_1 = \mathbf{G}\,\mathbf{f}$$

where **f** is the spatial load distribution vector. The first Ritz vector is, then, mass-normalized as:

$$\mathbf{r}_1 = \frac{\bar{\mathbf{r}}_1}{\left[\bar{\mathbf{r}}_1^T\, \mathbf{M}\, \bar{\mathbf{r}}_1\right]^{\frac{1}{2}}}$$

Subsequent Ritz vectors can be recursively generated as follows:

$$\bar{\mathbf{r}}_s = \mathbf{G}\mathbf{M}\,\mathbf{r}_{s-1}$$

The linear independence of Ritz vectors is achieved using the Gram-Schmidt orthogonalization:

$$\tilde{\mathbf{r}}_s = \bar{\mathbf{r}}_s - \sum_{t=1}^{s-1}\left(\mathbf{r}_t^T\, \mathbf{M}\, \bar{\mathbf{r}}_s\right)\mathbf{r}_t$$

Finally, the current Ritz vector is mass-normalized:

$$\mathbf{r}_s = \frac{\tilde{\mathbf{r}}_s}{\left[\tilde{\mathbf{r}}_s^T\, \mathbf{M}\, \tilde{\mathbf{r}}_s\right]^{\frac{1}{2}}}$$

AN EXPERIMENTAL BRIDGE MODEL

For this study, a grid-type bridge model was constructed and tested at the Hyundai Institute of Construction Technology (HICT), Korea (see Figure 1). The steel bridge model consists of two parallel girders and six evenly spaced cross beams connecting the two girders. The girders are steel rectangular tubes and the cross beams are C-shape members. Using impact excitations, we extract Ritz and modal vectors from the vibration response of this test structure. The sensors measure the vertical accelerations at the twelve joint locations as shown in Figure 1. Impulse excitation is also applied to every measurement point by an impact hammer. The rational polynomial technique [3] is employed to extract the first six natural frequencies and the corresponding modal vectors from the recorded FRFs.

A finite element (FE) model for the grid type bridge structure is constructed using twenty three-dimensional beam elements. Since the accelerometers measure only the vertical movement of the structure, the lateral DOFs are not included in the analytical model. Therefore, each node of an element has two translational DOFs and three rotational DOFs. The model has a total of 64 DOFs including four rotational DOFs at the boundary. Both ends of the beam are modeled as simple pinned connections. After some minor modifications made on the analytical model based on the recorded test data, the relative errors of the first six natural frequencies between the analytical model and the test structure fall within 4%.

Figure 1: An overview of a grid-type bridge structure

EXPERIMENTAL VERIFICATION

The new procedure for extracting Ritz vectors is demonstrated using the vibration data obtained from the test structure. Figure 2 compares the Ritz vectors estimated by the flexibility-based method with the corresponding Ritz vectors computed from the FE model. Imaginary point loads are simultaneously applied to nodes 2 and 13 (upward point load at node 2 and downward point load at node 13), and the corresponding Ritz vectors are generated. A complete flexibility matrix, including the modal and residual flexibility matrices estimated from the measured FRFs, is employed for the extraction of Ritz vectors. Figure 2 shows a good agreement between the analytical and experimental Ritz vectors. The first Ritz vector is equivalent to a static deflection pattern observed when the unit loads are applied at nodes 2 and 13. Note that the discrepancy between the analytical and experimental Ritz vectors increases for higher Ritz vectors.

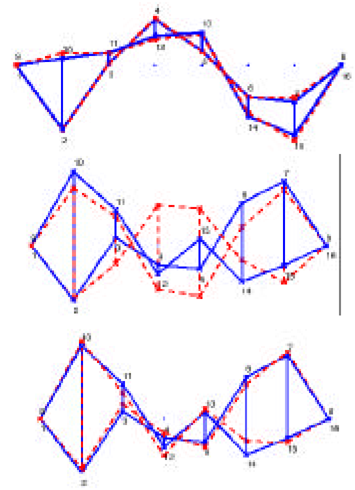
To investigate the influence of the residual flexibility, Figure 3 shows the experimental Ritz vectors obtained by using the measured modal flexibility alone. By comparing Figure 2 and Figure 3, it can be seen that the inclusion of the residual term significantly improves the accuracy of higher Ritz vectors. Table 1 shows a quantitative comparison using the Modal Assurance Criterion (MAC) value defined as follows:

$$MAC(i,j) = \frac{\left(\mathbf{r}_i^T \mathbf{M} \hat{\mathbf{r}}_j\right)^2}{\left(\mathbf{r}_i^T \mathbf{M} \mathbf{r}_i\right)\left(\hat{\mathbf{r}}_j^T \mathbf{M} \hat{\mathbf{r}}_j\right)}$$

where and are the analytical and experimental Ritz vectors, respectively.

Table 1: MAC values between analytical and experimental Ritz vectors

G	i for MAC (i, i)					
	1	2	3	4	5	6
G_m+G_r	0.9947	0.9930	0.9971	0.9956	0.9754	0.9720
G_m	0.9946	0.9930	0.9876	0.9099	0.3932	0.0211



(a) Ritz vector 4

(b) Ritz vector 5

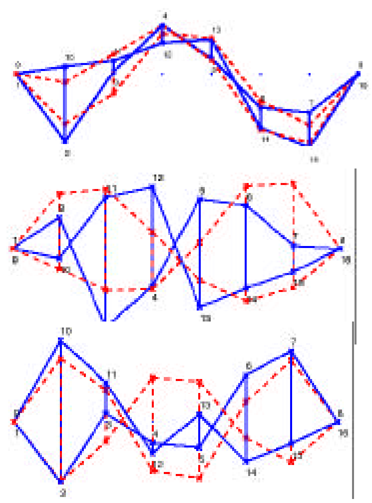
(c) Ritz vector 6

Figure 2: Comparison of analytical and experimental Ritz vectors (using a complete flexibility)

Similar results are obtained from comparing the experimental and analytical Ritz vectors when point loads are applied at nodes 2 - 7 and 10 - 15, respectively. When the measured modal flexibility is employed alone for the extraction procedure, the MAC values between the analytical and experimental Ritz vectors fall below 0.8 after the third Ritz vectors. The results indicate that the inclusion of the residual flexibility matrix can significantly improve the accuracy of the estimated Ritz vectors.

SUMMARY AND DISCUSSIONS

In this study, a new procedure has been proposed to extract load-dependent Ritz vectors using a statically complete flexibility matrix. First, measured modal flexibility and residual flexibility matrices are estimated from measured FRFs. Then, Ritz vectors are recursively generated using the measured flexibility matrix. The procedure is successfully demonstrated using an experiment of a grid-type bridge structure. Particularly, the influence of the residual flexibility on the extracted Ritz vectors is investigated. The proposed method has at least two advantages over the state-space method proposed by Cao and Zimmerman [1]: (1) the inclusion of the residual flexibility significantly improves the accuracy of the estimated Ritz vectors, and (2) the proposed method is able to generate Ritz vectors from any arbitrary load patterns.



(a)Ritz vector 4

(b)Ritz vector 5

(c) Ritz vector 6

Figure 3: Comparison of analytical and experimental Ritz vectors (using only a modal flexibility)

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EDUARDO MIRANDA JOINS NEW DESIGN PROGRAM

The Blume Center is pleased to announce that **Eduardo Miranda** joined the Stanford faculty in January as an Assistant Professor in the newly formed Design/Construction Integration Program. The DCI program (jointly administered by the Structural Engineering and Geomechanics Program and the Construction Engineering and Management Program) prepares students for multidisciplinary collaborative teamwork in an integrated design and construction process. The program extends a student's design or construction background with core courses in each of these areas and develops the background needed to understand the concerns and expertise of the many project stakeholders. It includes a comprehensive project-based learning experience.

Miranda received his Civil Engineering degree from the National Autonomous University of Mexico (UNAM) and his M.S. and Ph.D. degrees from U.C. Berkeley, where he developed seismic criteria for the evaluation and upgrading of existing structures. After completing his Ph.D., Miranda worked on the development and evaluation of various simplified methods to estimate the maximum response of nonlinear structures subjected to earthquakes. Prior to joining the faculty at Stanford, he taught earthquake engineering at UNAM and performed research at the National Center for Disaster Prevention. He also worked as a consulting structural engineer for six years specializing in earthquake engineering and seismic risk. At Stanford, Miranda intends to focus on design and construction integration and on performance-based design. He will teach a graduate level course in design and construction integration in the Spring Quarter.

RESEARCH SPOTLIGHT, Continued from page 3

ACKNOWLEDGMENT

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